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The Critical Radius Effect with a Variable Heat Transfer Coefficient

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Consider a solid object surrounded by a fluid. Heat transfer to or from the object requires a conduction-convection balance at the surface of the object. If the object has a curved surface (such as a circular cylinder or a sphere) and is smaller than a certain critical size, then adding insulation to the surface of the object has the effect of increasing rather than decreasing the heat transfer. This phenomenon, commonly called the critical radius effect, has been known for some time (for example, see Kreith, 1958), and the classical results for the critical radius are

$$r_{\text{critical}} = \begin{cases} \dfrac{k}{h} & \text{for a circular cylinder} \\[2ex] \dfrac{2k}{h} & \text{for a sphere} \end{cases} \tag{1}$$

Equation (1) requires that both k and h be constants, and whereas k is a physical property and can logically be taken to be constant, we know that h depends on various parameters (including the radius of the object) and in general is not a constant. Sparrow (1970) developed the formulas for the critical radius for cylinders and spheres for the case where h is given by

$$h = H\, r^{-m}\, (T_o - T_\infty)^n \tag{2}$$

where m and n are both $\geqq 0$, and H is a constant. In this situation, the formula for the critical radii becomes

$$r_{\text{cylinder}} = \frac{\left[\left(\dfrac{1-m}{1+n}\right)\left(\dfrac{k}{H}\right)\right]^{1/(1-m)}}{(T_o - T_\infty)^{n/(1-m)}} \tag{3}$$

$$r_{\text{sphere}} = \frac{\left[\left(\dfrac{1-m/2}{1+n}\right)\left(\dfrac{2k}{H}\right)\right]^{1/(1-m)}}{(T_o - T_\infty)^{n/(1-m)}} \tag{4}$$

Equation (2) is accurate only within the turbulent regime. However, correlations currently exist which span both laminar and turbulent free convection. Churchill and Chu (1975) have developed the following relationship for free convection around a horizontal cylinder:

$$h = \frac{k_f}{2r}\left\{0.60 + 0.387\left[\frac{GrPr}{\left[1+\left(\dfrac{0.559}{Pr}\right)^{9/16}\right]^{16/9}}\right]^{1/6}\right\}^2 \tag{5}$$

for $10^{-5} < GrPr < 10^{12}$, where Pr is the dimensionless Prandtl number of the surrounding fluid, and Gr is the dimensionless Grashof number. Yuge (1960) has developed the following equation for free convection around spheres:

$$h = \frac{k_f}{2r}\,(2 + 0.392\, Gr^{1/4}) \tag{6}$$

for $1 \leqq Gr \leqq 10^5$.

Equations (5) and (6) can be generalized in a manner similar to Equation (2), resulting in

$$h = \left[\frac{C_1}{\sqrt{r_o}} + C_2(T_o - T_\infty)^{1/6}\right]^2 \tag{7}$$

for a horizontal cylinder and

$$h = \frac{C_3}{r_o} + C_4(T_o - T_\infty)^{1/4}\, r_o^{-1/4} \tag{8}$$

for a sphere. The influence of the laminar regime gives Equations (7) and (8) a considerably different form from Equation (2). Equations (5) and (6) can now be used in the heat balance at the object's surface, and the critical radius can be determined by setting

$$\frac{dq}{dr} = 0 \quad \text{at} \quad r = r_{\text{critical}} \tag{9}$$

and solving for r_{critical}. When this is done, one gets (for details, see Balmer and Strobusch, 1977):

Cylinder

$$r_{\text{critical}} = \left(\alpha + \frac{a}{b}\right)^2 \tag{10}$$

where α is one of the real roots of the cubic equation:

$$\alpha^3 + \frac{a}{b}\alpha^2 + \left(\frac{3a^2 - k}{b^2}\right)\alpha + \left(\frac{a}{b}\right)^3 = 0 \tag{11}$$

and

$$a = 0.4243\,(k_f)^{1/2} \tag{12}$$

$$b = 0.2737\,(k_f)^{1/2}\left[\frac{8g\beta(T_o - T_\infty)Pr/(\nu^2)}{\left[1+\left(\dfrac{0.559}{Pr}\right)^{9/16}\right]^{16/9}}\right]^{1/6} \tag{13}$$

Sphere

$$r_{critical} = \frac{1}{C_5}\left[\left(\frac{7k}{8} - k_f\right) \pm \left[\left(\frac{7k}{8} - k_f\right)^2 + k_f(k - k_f)\right]^{1/2}\right]^{4/3} \quad (14)$$

where

$$C_5 = 0.196\, k_f \left[\frac{8g\beta(T_o - T_\infty)}{\nu^2}\right]^{1/4} \quad (15)$$

Equations (10) and (14) reduce to the form of Equations (3) and (4), only when the additive constants in Equations (5) and (6) (0.6 and 2.0, respectively) are negligible when compared to the term containing the Grashof number. Equations (10) and (14) are, therefore, somewhat more general than Equations (3) and (4).

Simmons (1976) used Equation (2) in determining the critical radius for combined convection and radiation. However, the algebraic simplification that resulted from the use of Equation (2) in his analysis does not occur with the use of Equations (7) and (8). Consequently, the use of these more general convective heat transfer coefficient correlations in Simmons' analysis does not produce an explicit solution for the critical radius in the case of combined convection and radiation.

NOTATION

C_1, C_2, C_3, C_4 = constants in Equations (7) and (8)
Gr = Grashof number
h = convective heat transfer coefficient ($Wm^{-2}\,°C^{-1}$)
H = constant in Equation (2)
k = thermal conductivity of object ($Wm^{-1}\,°C^{-1}$)
k_f = thermal conductivity of surrounding fluid ($Wm^{-1}\,°C^{-1}$)
m, n = constants in Equation (2) (≥ 0)
Pr = Prandtl number
q = heat transfer (W)
r = radius of the object (m)
T_o = surface temperature of the object (°C)
T_∞ = fluid temperature (°C)

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A Correlation for Solid Friction Factor in Vertical Pneumatic Conveying Lines

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Flowing gas-solids suspensions, commonly known as pneumatic conveying or pneumatic transport, have been widely practiced in the industry in loading and unloading of dry bulk materials and in distributing raw materials from the storage bins to the chemical reactors. Smooth operation of industrial chemical plants depends on reliable operation of these blood lines to supply the raw materials and to remove the wastes. Pneumatic transport is one of the known economic and feasible methods for performing these functions at low and medium pressure operations (up to ~25 atm). Chemical reactions and physical operations such as drying can also be carried out in flowing gas-solids suspensions known as entrained bed reactors. Despite its importance, design of a pneumatic transport system remains an art rather than a science (Leung and Wiles, 1976). This paper will focus the discussion on only one aspect of the pneumatic transport, the frictional loss caused by solid particles in dilute phase vertical transport.

PRESSURE DROP IN A DILUTE PHASE VERTICAL PNEUMATIC CONVEYING LINE

There are numerous correlations available in the literature for predicting pressure drop in dilute phase vertical pneumatic conveying as recently reviewed by Leung and Wiles (1976). The general recommended approach is to consider that the total pressure drop consists of three individual contributions due to acceleration, gravity, and wall friction:

$$\Delta P_T = \Delta P_A + \Delta P_S + \Delta P_F \quad (1)$$

The acceleration loss ΔP_A can be evaluated as proposed by Yang and Keairns (1976)

$$\Delta P_A = \int_o^l \rho_p(1-\epsilon)g\,dl + \int_o^l \frac{2f_g\,\rho_f U_f^2}{D}\,dl + \int_o^l \frac{f_p\,\rho_p(1-\epsilon)U_p^2}{2D}\,dl + [\rho_p(1-\epsilon)U_p^2]_{at\ l} \quad (2)$$

Beyond the acceleration region, the static head term can be expressed by

$$\Delta P_S = \rho_p(1-\epsilon)g \cdot L \quad (3)$$

The friction term ΔP_F is often separated into two terms due separately to fluid alone and to the effect of solid particles:

$$\Delta P_F = \Delta P_{Fg} + \Delta P_{FS} \quad (4)$$

The friction due to conveying fluid alone is usually defined

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